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IMPERFECT MARKET-MAKER COMPETITION, HETEROGENEOUS EXPECTATIONS, AND THE FAVOURITE-LONGSHOT BIAS IN WAGERING MARKETS

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In the study of wagering markets, it is generally the case that the objective probabilities of various contestants (horses, teams, etc.) winning do not match those implied by the betting. More often than not favourites are underbet and longshots overbet, although some studies have found the reverse. We offer an explanation in the case where there is imperfect competition among book-makers and heterogeneous expectations among bettors.

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INTRODUCTION

In betting markets, there is extensive empirical support for the *favourite-longshot bias*, the phenomenon that favourites are underbet and longshots are overbet (the "usual" bias). The interested reader is referred to Raymond Sauer (1998) and Richard Thaler and William Ziemba (1988) for good summaries of this evidence. While most studies have found the usual bias, Kelly Busche and Christopher Hall (1988) and Linda Woodland and Bill Woodland (1994) find the bias in reverse direction. The firm conclusion is that there is a bias, although, in some markets, a reverse bias has been observed.

The effort to explain this phenomenon has been ongoing for several decades. Most theories turn on one or more of the following characteristics:

1. Bettor Risk Preferences;
2. Information (both asymmetric and heterogeneous information); and
3. Market Micro-Structure (pari-mutuel versus a book-maker market).

For the most part, the explanations based on bettor risk preferences take the position that bettors are risk lovers. This line of research would include the work of Martin Weitzman (1965), Mukhtar Ali (1977), Richard Quandt (1986), and Antti Kanto, Gunnar Rosenqvist, and Arto Suvas (1992). More recently Joseph Golec and Maurry Tamarkin (1998) have suggested that bettors prefer return skewness

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rather than risk and Kelly Busche and David Walls (2000) argue that bias may be an empirical issue related to even higher moments of the return distribution.

The rest of the important contributions turn on Information and/or Market Micro-Structure. Ali (1977) studies a two-horse race where bettors have heterogenous expectations and the betting mechanism is pari-mutuel. He shows that the usual bias will obtain. Stephen Blough (1994) extends Ali's model to the case of an *n*-horse race and finds that, under the condition of symmetric heterogenous expectations, the bias will emerge. William Hurley and Lawrence McDonough (2005) study the case of sequential pari-mutuel betting with heterogeneous expectations and show that both the usual and reverse biases are possible depending on the distributions of bettor beliefs. Their sequence of bettors produces a stochastic process of pari-mutuel odds and the steady-state behavior of this sequence is examined. In contrast, Rob Feeney and Stephen King (2001) and Takahiro Watanabe (1997) model sequential pari-mutuel betting as a game.

Another set of models turns on asymmetric information. These models assume that there is a class of bettor which is more informed about the outcome of the contest and would include the work of Hyun Shin (1992), Hurley and McDonough (1995, 1996), Leighton Vaughan Williams and David Paton (1998) and Micheal Cain, David Law, and David Peel (2003). Hurley and McDonough employ a pari-mutuel mechanism; the others study a book-maker market.

As for Market Micro-Structure, it is well known that the institutional characteristics of a market are important in asset price formation. Betting markets are no exception. These markets are organized in two main ways. In one, trading follows the pari-mutuel mechanism where the final odds of a bet are not known until the close of betting. In the other, book-makers stand ready to take bets at fixed odds. The two are fundamentally different.

By way of summary, the following table classifies the literature along the dimensions of Information and Market Micro-Structure:

	Asymmetric Information	*Heterogeneous Information*
Pari-mutuel Mechanism	Hurley/McDonough (1995) Hurley/McDonough (1995)	Ali (1977) Blough (1994) Feeney and King (2001) Hurley/McDonough (2005) Watanabe (1997)
Book-Maker	Shin (1992) Williams/Paton (1998) Cain/Law/Peel (2003)	

Note the gap in the *Book-Maker/Heterogeneous Information* quadrant. Hence, the purpose of this paper is to explore the case where betters have heterogeneous beliefs and the book-maker market is imperfectly competitive.

We consider the case of a single book-maker behaving as a profit-maximizing monopolist. For our two-horse race, bettor expectations on the favourite horse are modelled as drawings from a probability distribution centered on the favourite's objective probability of winning. That is, the mean of the distribution is equal to the favourite's objective probability of winning. Each bettor examines the prices set by the book-maker (contingent claims paying \$1) and his beliefs to determine which horse to bet. Given these assumptions, we show that the usual and reverse biases can occur depending on the distribution of bettor beliefs. To our knowledge, this model is the first to produce the bias in both directions. We will show that this direction depends critically on the distribution of bettor beliefs.

We contrast this result to the case where book-makers behave as Bertrand competitors and show that there is no bias regardless of the nature of bettor expectations. We conclude that a real-world book-maker market is likely to lie somewhere between these two extremes of monopoly and pure competition. That is, if there is not some degree of book-maker cooperation, there might well be some other market imperfection such as bettor search cost which has the same effect. Consequently, our model predicts that there will be a bias, either reverse or usual, and the size of the bias will depend fundamentally on the distribution of bettor beliefs.

A SINGLE MARKET-MAKER

Consider a horse race where there are only two horses - a *Favourite* and a *Longshot*. The true probability that the Favourite wins is $p_F > 1/2$; the probability that the Longshot wins is $p_L = 1 - p_F$. We assume our book-maker (market-maker, MM) knows these true probabilities. The MM is prepared to take bets on the both horses. For a price of π_F, a bettor is entitled to \$1 if the Favourite wins and 0 otherwise. For a price of π_L, a better gets \$1 if the Longshot wins and 0 otherwise. Hence the MM offers two simple contingent claims.

We model bettor behavior as follows. There are N possible bettors where N is large. We assume bettor i's expectation, $\tilde{p}_i$, is formed by a random drawing from a distribution with density function $g(\cdot)$ and a corresponding cumulative density function $G(\cdot)$. Hence our bettors have heterogeneous beliefs for a particular race, but we insist that they be unbiased. Mathematically we require that

$$(1) \qquad E(\tilde{p}_i) = p_F \quad \text{for all } i.$$

That is, for a large number of betters, we have that

$$(2) \qquad \frac{\tilde{p}_1 + \tilde{p}_2 + \cdots + \tilde{p}_N}{N} \xrightarrow{\text{pr}} p_F.$$

Equivalently, the average of bettor beliefs gets arbitrarily close to p_F as N gets large. Note that each bettor makes mistakes on individual races, but over a large number of identical races, the average of his estimates converges on the Favourite's objective probability of winning.

We assume that bettors wager simultaneously and with hubris. That is, each makes the assumption that he/she is the only one among the throng of bettors who is able to interpret the available information correctly. Hence, each takes the MM prices as given and does not update his/her beliefs on the basis of these prices. Moreover, the MM posts his/her prices once at the beginning of betting and does not change them as betting progresses. Even if we were to assume sequential betting, the MM could not learn anything from the betting since bettor expectations are assumed to be independent.

For the moment we are going to make a second assumption that $g(\cdot)$ is symmetric about p_F:

(3) $$g(p_F - x) = g(p_F + x) \quad \text{for all } x > 0.$$

This implies that g has a median at p_F:

(4) $$G(p_F) = \frac{1}{2}.$$

We also assume that bettors will wager only if the expected return of a bet is nonnegative. For instance, consider bettor i. His expected return on the Favourite is

(5) $$r_i^F = 1 \cdot \tilde{p}_i - \pi_F = \tilde{p}_i - \pi_F$$

and on the Longshot,

(6) $$r_i^L = 1 \cdot (1 - \tilde{p}_i) - \pi_L = 1 - \tilde{p}_i - \pi_L.$$

He will choose the wager which gives the highest return provided this return is nonnegative. We can show that the Favourite will be bet if

(7) $$\tilde{p}_i \geq \pi_F,$$

and the Longshot if

(8) $$\tilde{p}_i \leq 1 - \pi_L.$$

The MM's expected profit is the sum of two parts, a profit on Favourite betting, and one on Longshot betting. The MM's expected revenue on the Favourite is $\pi_F(1 - G(\pi_F))N$ and his payout is \$1 for each of the $(1 - G(\pi_F))N$ bettors on the Favourite. Since this payout happens with probability p_F, the MM's expected profit on the Favourite is

(9) $$N\{\pi_F(1 - G(\pi_F)) - (1 - G(\pi_F))p_F\} = N\{(\pi_F - p_F)(1 - G(\pi_F))\}.$$

A similar argument on the Longshot gives an expected profit of

(10) $$N(\pi_L + p_F - 1)G(1 - \pi_L).$$

Hence total profit is

(11) $$\varphi(\pi_F, \pi_L) = N\{(\pi_F - p_F)(1 - G(\pi_F)) + (\pi_L + p_F - 1)G(1 - \pi_L)\}.$$

Note that we have not included a cost function. Our omly assumption in this regard is that revenues exceed the MM's cost.

The first-order conditions for maximizing $\varphi(\pi_F, \pi_L)$ are

(12a) $$(\pi_F - p_F)g(\pi_F) + G(\pi_F) - 1 = 0$$

(12b) $$(\pi_L - p_L)g(1 - \pi_L) - G(1 - \pi_L) = 0.$$

For some distributions, these conditions are easily solved. For instance if bettor expectations are uniformly distributed on the interval

(13) $$\left[p_F - 1/2 \cdot \delta, p_F + 1/2 \cdot \delta\right],$$

the optimal prices are

(14) $$\begin{aligned} \pi_F^* &= p_F + \delta/4 \\ \pi_L^* &= p_L + \delta/4. \end{aligned}$$

It is straightforward to show that these prices satisfy the second-order conditions. Note that the off-diagonal terms of the Hessian are 0 and hence it is sufficient to check that

(15) $$\frac{\partial^2 \varphi}{\partial \pi_F^2} < 0 \quad \text{and} \quad \frac{\partial^2 \varphi}{\partial \pi_L^2} < 0.$$

Regarding the solution in (14), note that δ characterizes the degree of heterogeneity in bettor beliefs. The larger δ is, the greater the degree of differences in belief. Similar to Blough's finding, note that the MM's markup is

(16) $$\pi_F^* + \pi_L^* = p_F + \delta/4 + p_L + \delta/4 = 1 + \delta/2$$

so that, the higher the degree of heterogeneity, the larger the markup. Note as well that MM profit at the optimal prices, $\varphi^* = N\delta/8$, increases with the degree of heterogeneity.

We now show that the prices which satisfy the first-order conditions give rise to the favourite-longshot bias. To do so we first need to prove that optimal prices have the same form as those in (14).

Lemma. Suppose bettor expectations are given by the distribution $g(\cdot)$ and that $g(\cdot)$ is symmetric about p_F. Then the MM's optimal prices, if they exist, have the form

(17) $$\begin{aligned} \pi_F^* &= p_F + \Delta \\ \pi_L^* &= p_L + \Delta \end{aligned}$$

for $\Delta > 0$.

Proof. Suppose that $\pi_F^* = p_F + \Delta$ solves (12a). Then substituting directly, we have that

$$(18) \qquad \Delta \cdot g(p_F + \Delta) + G(p_F + \Delta) - 1 = 0.$$

But by the symmetry of $g(\cdot)$, $g(p_F + \Delta) = g(p_F - \Delta)$ and $1 - G(p_F + \Delta) = G(p_F - \Delta)$. Substituting these into (19) gives

$$(19) \qquad \Delta \cdot g(p_F - \Delta) - G(p_F - \Delta) = 0.$$

Substituting $\Delta = \pi_L - p_L$ and $p_F = 1 - p_L$ gives

$$(20) \qquad (\pi_L - p_L) \cdot g(1 - p_L - \Delta) - G(1 - p_L - \Delta) = 0$$

or

$$(21) \qquad (\pi_L - p_L) \cdot g(1 - \pi_L) - G(1 - \pi_L) = 0.$$

Therefore $\pi_L^* = p_L + \Delta$ solves (12b). Hence $\pi_L^* = p_L + \Delta$ is optimal. A similar argument beginning with (12b) shows that $\pi_F^* = p_F + \Delta$ is also optimal. Note that Δ must be positive to assure that profits are non-negative. And the proof is complete.

The optimal prices are clearly biased since

$$(22) \qquad \frac{\pi_F^*}{\pi_L^*} = \frac{p_F + \Delta}{p_L + \Delta} < \frac{p_F}{p_L}$$

and the Favourite is underbet. But what we are really interested in is the relationship of the objective and subjective probabilities. In a pari-mutuel setting we could infer the subjective probability on the Favourite, θ_F, from the ratio of MM revenues on the Favourite to total revenues:

$$(23) \qquad \theta_F = \frac{\pi_F \cdot N(1 - G(\pi_F))}{\pi_F \cdot N(1 - G(\pi_F)) + \pi_L \cdot NG(1 - \pi_L)}.$$

Similarly, the subjective probability on the Longshot is defined

$$(24) \qquad \theta_L = \frac{\pi_L \cdot NG(1 - \pi_L)}{\pi_F \cdot N(1 - G(\pi_F)) + \pi_L \cdot NG(1 - \pi_L)}.$$

But in bookmaker markets, we are not usually able to observe the dollar betting volume on each horse. Normally subjective probabilities are inferred from relative prices:

$$(25) \qquad \theta_F^{\pi} = \frac{\pi_F}{\pi_F + \pi_L}$$

$$(26) \qquad \theta_L^{\pi} = \frac{\pi_L}{\pi_F + \pi_L}.$$

In the following proposition we will show that

$$\theta_F = \theta_F^\pi \quad \text{and} \quad \theta_L = \theta_L^\pi. \tag{27}$$

That is, the two approaches are equivalent.

Proposition. Suppose bettor expectations are given by the distribution $g(\cdot)$ and that $g(\cdot)$ is symmetric about p_F. Then, at the solution

$$\begin{aligned} \pi_F^* &= p_F + \Delta \\ \pi_L^* &= p_L + \Delta, \end{aligned} \tag{28}$$

the usual bias obtains:

$$\theta_F < p_F \quad \text{and} \quad \theta_L > p_L. \tag{29}$$

Proof. We first show that, at the optimum prices, the expected number of bettors on the Favourite, N_F, is equal to the expected number of bettors on the Longshot, N_L:

$$N_F = N(1 - G(\pi_F)) = N(1 - G(p_F + \Delta)) = NG(p_F - \Delta) = N_L$$

Hence

$$\theta_F = \frac{N_F \pi_F}{N_F \pi_F + N_L \pi_L} = \frac{\pi_F}{\pi_F + \pi_L}. \tag{30}$$

Substituting (17) gives

$$\theta_F = \frac{\pi_F}{\pi_F + \pi_L} = \frac{p_F + \Delta}{1 + 2\Delta} \tag{31}$$

which is always less than p_F for $\Delta > 0$ and $p_F > 1/2$. Similarly, we have that

$$\theta_L = \frac{\pi_L}{\pi_F + \pi_L} = \frac{p_L + \Delta}{1 + 2\Delta} \tag{32}$$

and this is greater than p_L when $p_L < 1/2$. And the proof is complete.

It is worth noting that a common empirical finding in wagering markets is that book-makers try to balance the betting capital among all horses and this can be interpreted as a risk averse strategy. (John Fingleton and Patrick Waldron (1996)). Our model's results are consistent with this finding. Moreover, if the Favourite wins, his net expected revenues are

$$R_F = N_F \pi_F^* + N_L \pi_L^* - N_F = N_F\left(\pi_F^* + \pi_L^* - 1\right) = 2\Delta N_F \tag{33}$$

and if the Longshot wins, they are

$$R_L = N_F \pi_F^* + N_L \pi_L^* - N_L = N_F\left(\pi_F^* + \pi_L^* - 1\right) = 2\Delta N_F = R_F. \tag{34}$$

Also note that, at the optimum,

$$(35) \qquad \pi_F^* + \pi_L^* = p_F + \Delta + p_L + \Delta = 1 + 2\Delta.$$

Hence, the book-maker's expected net revenues are always positive. Our result is that risk neutral book-makers maximize profits by balancing the books in the case where bettor beliefs follow a symmetric distribution.

THE REVERSE BIAS IS POSSIBLE

Suppose that bettor beliefs are drawn from the density

$$(36) \qquad g(x) = \frac{2(x-a)}{(b-a)^2} \quad a \leq x \leq b.$$

This density is linear and upward sloping over the complete interval. Its expected value is

$$(37) \qquad E(X) = \frac{1}{3}a + \frac{2}{3}b$$

and its cumulative density is

$$(38) \qquad G(x) = \left(\frac{x-a}{b-a}\right)^2 \quad a \leq x \leq b.$$

Consider an example where $a = .3$ and $b = .9$. At these parameter values, the objective probability that the Favourite wins is

$$(39) \qquad p_F = E(X) = \frac{1}{3}(.3) + \frac{2}{3}(.9) = .7.$$

The optimal prices are

$$(40) \qquad \begin{aligned} \pi_F^* &= 0.8045 \\ \pi_L^* &= 0.4333 \end{aligned}$$

and these satisfy the second-order conditions. The subjective probability on the Favourite at these prices is

$$(41) \qquad \theta_F = .734.$$

This is the reverse bias since the subjective probability on the Favourite, .734, exceeds its objective probability, .7. Hence it is possible to get the reverse bias in cases where the distribution of bettor expectations is nonsymmetric.

CONCLUSION

It is difficult to conceive of competing MMs as Cournot competitors. Hence we consider the case where they are Bertrand competitors who compete fiercely on price to the point where each makes zero profit. We can show that the Bertrand outcome is $\pi_F = p_F$ and $\pi_L = p_L$, and profits for both MMs are zero at these prices. Moreover, at these prices, there is no bias, that is, $\theta_F = p_F$.

The nature of real-world competition in book-maker markets suggests that reality is probably somewhere between pure monopoly and the Bertrand outcome. But the bias is 0 only at the Bertrand outcome. Hence, any imperfection in book-maker competition will produce a bias. The direction of this bias will depend critically on the distribution of bettor expectations. Our model suggests that it would be highly unlikely that we not observe a favourite-longshot bias in these markets.

An explanation of the longshot-favourite bias should attempt to reconcile three generally observable facts:

1. the bias is usually positive but may be negative in rare instances;
2. book-makers tend to balance their books; and
3. individual bettors have different beliefs about the outcome of the event they are betting on.

The model developed here either assumes or implies these features. By implication, attribution of any empirical measure of bias to informational asymmetry must be interpreted with caution. Indeed there may be no way to differentiate between informational asymmetry and imperfect competition as sources of bias.

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