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INVESTING IN MARCH MADNESS: AN EXAMINATION OF THE RELATIONSHIP BETWEEN SPORTS BETTING AND PORTFOLIO CONSTRUCTION

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ABSTRACT

This study focuses on three basic ideas: (1) sports betting should be considered an asset class; (2) sports betting contests, with two teams or contenders, are well described by a recently introduced probability distribution, the generalized Poisson binomial (GPB) distribution; and (3) Modern Portfolio Theory (MPT), Post-Modern Portfolio Theory (PMPT), and the Kelly criterion can be applied to yield optimal risk-return portfolios in such two contender environments. For the PMPT application, a unique quadratic-binary programming model is developed. March Madness data, based on the NCAA men's annual championship basketball tournament, will provide examples of these portfolio theory approaches.

Keywords: sports betting, generalized poisson binomial distribution, modern portfolio theory, post-modern portfolio theory, Kelly criterion

1 INTRODUCTION

Many, if not most, sports contests involve two opposing contenders, whether they be two opposing teams or two opposing individuals, resulting in a binary outcome of a win or loss for each party. Recently, Zhang et al. (2018) developed a new probability distribution, which they suggest may be used for a variety of technical applications. It is shown here that this new distribution also precisely models sports contests with two opposing contenders. As seen below, it has been suggested, and is further argued here, that sports betting has many of the characteristics of an asset class. This new distribution, then, opens the door to

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developing sports betting investment portfolios with optimal properties. While optimal financial portfolio approaches have been discussed for many years, this is the first research to develop such portfolios for the common and important case of two-party sports contests.

Thus, this research demonstrates that (1) sports betting should be considered an asset class; (2) sports betting contests, with two teams or contenders, in a binary outcome, are well described by a recently introduced probability distribution, the generalized Poisson binomial (GPB) distribution; and (3) Modern Portfolio Theory (MPT), Post-Modern Portfolio Theory (PMPT), and the Kelly criterion can be applied to yield optimal risk-return portfolios in such two contender environments. For the PMPT application, a unique quadratic-binary programming model is developed. March Madness data, based on the NCAA men's annual championship basketball tournament, will provide examples of these portfolio theory approaches.

The analogy between gambling and investment in financial instruments has been noted by Gray and Gray (1997), Thorp (2006), and Gomber et al. (2008). In what follows, this analogy is highlighted and explored. It is suggested that there is a spectrum with two poles to describe the motivation for individuals to gamble, with a continuum between two poles. At one pole are those who see gambling as an entertainment activity. Such individuals will be labelled as entertainment-driven bettors. At the opposite pole are those who see gambling as an investment opportunity and these individuals will be labelled as investment-driven bettors. A new model that develops optimal portfolios and efficient frontiers for gambling on contests between two adversaries is developed based on a recently introduced probability distribution.

Ignatin (1984) developed two general characterizations for sports betting activities, consumption and investment. Consumption would include those who are seeking to increase the utility or intrinsic satisfaction a person achieves while watching a sporting event (Ignatin, 1984). Most often this will include bettors who simply enjoy making the experience of watching a game more interesting; these people are not wagering as part of an investment strategy (entertainment-driven bettors). The most common reason that entertainment-driven people were involved in these activities was that people wanted "to have a good time" as discussed in the study *Gambling in America: Final Report*. (US Commission on the Review of the National Policy Toward Gambling, 1976, p. 67). Jenkinson et al. (2018) conducted a study on young male bettors in 2018 which determined the motivations for participating in sports betting: that it makes the sporting event more interesting, the social aspect of it, the adrenaline rush, and the avoidance of boredom (Jenkinson et al., 2018). Entertainment-oriented betting is a common practice. However, there are a number of people who engage in sports betting with the expectation of generating an income stream through research and risk mitigation techniques. Investment-oriented gamblers refer to people who undertake these activities in order to create more wealth (Ignatin, 1984) while mitigating risk.

The methods utilized in this paper are oriented towards these investment-driven bettors. As two adversary sports contests are commonplace, for those who wish to intelligently invest in sports gambling, the models developed herein can help bring sports contests into a scientifically based financial decision process.

Gambling is defined as "placing something of value at risk in the hopes of gaining something of greater value" (Potenza et al., 2002, p. 721). Gambling in the United States has had an uneasy relationship with the law as it has been legally allowed in only certain areas. The federal Professional and Amateur Sports Protection Act of 1992 effectively banned sports betting across the United States, although four states were specifically exempted, namely, Delaware, Montana, Nevada, and Oregon. This law was overturned in the 2018 decision *Murphy v. National Collegiate Athletic Association* (138 S. Ct. 1461). Since this decision, there has been a very public and expanding sports betting industry, with companies such as DraftKings, Fanduel, BetOnline, and YouWager gaining prominence. DraftKings held its initial public offering in April 2020 as sports betting moved mainstream. The discussion of sports betting on a much broader scale is still a very new concept, and the research and data regarding sports betting are limited. Davis et al. (2018) defined three areas within sports betting that have been addressed in the literature. First, "psychological/behavioral factors influence the efficiency of sports betting markets" (Davis et al., 2018, p. 69) as referenced by *Asset Pricing and Sports Betting* (Moskowitz, 2021) and *Inefficient Pricing From Holdover Bias in NFL Point Spread Markets* (Foder et al., 2013); second, "mechanical or structural issues within sports betting markets" (Davis et al., 2018, 69) such as *Sports Betting as an Asset Class* (Gomber et al., 2008) and *Testing Market Efficiency: Evidence from the NFL Sports Betting Market* (Gray and Gray, 1997); and third, "potentially effective sports betting strategies including those that seek to utilize either behavioral factors or market asymmetries" (Davis et al., 2018, p. 69) such as *On Arbitrage and Market Efficiency: An Examination of NFL Wagering* (Burkey, 2005), *The Efficiency of Sports Betting Markets: An Analysis Using Arbitrage Trading within Super Rugby* (Buckle and Huang, 2018), and *Exploiting Sports Betting Using Machine Learning* (Hubacek et al., 2019). We seek to designate a fourth extension that would allow sports betting portfolio construction for the investment-driven bettor using modern financial and statistical theory. The beginnings of this approach may be found in *Markowitz Portfolio Theory for Soccer Spread Betting* (Fitt, 2009) which introduces some of these ideas. In 2021, the United States saw record commercial gaming revenue of $52.99 billion (American Gaming Association, 2022). Thus, it has become a significant economic activity.

There are a number of terms that are necessary to understand in the jargon of the betting world. The team or contender that is deemed most likely to win a matchup is referred to as the favourite. The team or contender that is not favoured to win is referred to as the dog (derivative of underdog). Money line bets are commonplace in American betting and are the odds used later in this paper. Simply put, the bettor wagers a specified amount. If the bettor wins, then a pre-agreed amount is returned to the bettor, in addition to the initial amount bet. If the bettor loses, then the initial bet is lost. The probability of winning is crucial to the bettor making a rational betting decision.

2 SPORTS BETTING AS AN ASSET CLASS

Graham and Dodd (1940, p. 63) define an investment "[a]s one which, upon thorough analysis, promises safety of principal and a satisfactory return." This principal sum will assume some form of risk (and a decrease in the liquidity of the principal) and thus requires a return in addition to his original payment. The satisfactory return is most commonly distributed in the forms of dividends, price appreciation, and interest payments. Graham and Dodd (1940, p. 107) clarified that the meaning of safety of principal is "protection against loss under all normal or reasonably likely conditions or variations." Hayes expands on the "safety of principal" by arguing that "it warns the investor that he may expect the market value of his portfolio to deteriorate below cost at any time" (1950, p. 390).

This concept of safety of principal "further makes clear that so long as no untoward event intervenes to vitiate his appraisal of the long-term prospects, he has all the essential elements of safety" (Hayes 1950, p. 390). Graham and Dodd (1940) published their work decades before Post-Modern Portfolio Theory would be introduced, but the terminology, "a satisfactory return" points to a return that is not simply positive but at a level considered to be acceptable by the investor; this idea is expanded upon with "minimum acceptable return" in Post-Modern Portfolio literature. An additional criterion encompasses the definition of investment: "An investment operation is one that can be justified on both qualitative and quantitative grounds" (Graham and Dodd 1940, p. 107). An alternate view of an investment is provided by Malkiel in his book, *A Random Walk Down Wall Street* in which he writes, "Investing [i]s a method... to gain profit in the form of reasonably predictable income (dividends, interest, or rentals) and/or appreciation over the long term" (Malkiel, 1973, p. 26). Normal investments are conducted through three main asset classifications as introduced by Robert Greer; these classifications are Capital Assets, Consumable/Transformable Assets, and Store of Value Assets (Greer, 1997).

"An asset class is a set of assets that bear some fundamental economic similarities to each other, and that have characteristics that make them distinct from other assets that are not part of that class" (Greer, 1997, p. 86). Gomber et al. (2008) created an argument for sports betting to be considered an investable asset. They describe a bet as a "leveraged product with the character of an option. The outcome is based on an underlying event which defines its maturity" (Gomber et al., 2008, p. 171). In this scenario, the bet would derive its value from a contest, and this event would determine the length of time for the bet. The sports bets discussed in this paper are short term; usually, the bet is placed either the day before or the day of the matchup. Investing is not limited to the traditional asset classes but includes a number of alternative asset classes. Recently, there have been new arguments for different items to be included as asset classes including Campbell (2008) arguing for art, Masset and Henderson (2010) arguing for wine, and Sontakke and Ghaisas (2017) arguing for cryptocurrencies, as well as sports betting by Gomber et al. (2008).

Thus, while these activities are riskier and more niche in nature, many consider them to be a form of investment. The draw of riskier investments (speculation) is the potential for large gains. Graham and Dodd referred to speculation as the assumption of risk that is implicit in a situation and thus must be borne by someone. They proposed two categories of speculation which include intelligent and unintelligent speculation. The former refers to the assumption of risk that appears justified after careful weighing of the pros and cons, whereas unintelligent speculation is defined as taking risk without adequate study of the situation (Graham and Dodd, 1940).

The relationship between investing and gambling exists not only within the structure of both practices but also within the psychology of those who utilize these practices. Much of the literature and cultural works portray gambling and investing as two very separate practices not to be intertwined. However, research into the relationship between the two practices is increasing and there does appear to be a relationship between investing and gambling. Arthur et al. (2016) explored the conceptual relationship between investing and gambling while specifically examining the empirical relationship between financial speculation and gambling. Jadlow and Mowen (2010) determined that investors and bettors overlap on five different personality traits while differing on three. Asch et al. (1982) examined racetrack betting and argued for the existence of an informed class of bettors. Generally, the research supports the view, as described in the introduction, of a continuum between the poles of entertainment-driven and investment-driven betting.

Chance and probability are most often associated with gambling; however, investment-driven sports betting requires analytical skill and strategy to win over the long term. This skill results from being able to interpret data in order to form an educated investment strategy. "Horse racing, sports betting and person to person games are types of gambling where a long term positive expected return occurs for a small number of knowledgeable and skilled gamblers" (Arthur et al., 2016, p. 582). In an efficient market, there are only two means through which a person can outperform the market, access to material non-public information, and superior analytical skills relative to his/her peers. Similarly, sports betting with material non-public information such as injuries, fixing of games, and other illegal activities offers benefits to those possessing the information. Having superior analytical skills may translate to better performance in sports wagering, specifically when there are a large number of inputs available to allow investors to make educated bets.

Some economists and psychologists have argued that gambling is irrational. Economists such as Ignatin (1984) argued that because of the law of diminishing marginal utility, the extra utility that can be won is smaller than the utility that can be lost when a bet is placed. This is a common view of the gambling world and is often correct in its assumptions. There is a difference between casino gambling and sports betting, but Ignatin (1984) did not differentiate between the two activities. Many casino games are driven strictly by luck; there is no skill or research that can be used to improve the chances of winning. Sports betting, on the contrary, involves skill and intelligence in the processing and application of available information.

Many similarities exist between investing and gambling; however, there exist differences. Traditionally, many assets (physical and financial) are held for a long period of time to capture the potential benefit of price appreciation. This period of time varies from months to years depending on the asset that is being held. Sports betting has a much shorter life span for the majority of mediums through which it can be undertaken. Most bets mature within a day of being placed. The bet is placed prior to the match, the match occurs, and the outcome is determined. The gain or loss is realized immediately upon completion of the event. In comparison with options, sports betting has an expiration date where the gain or loss will be realized, after which the bet has no value.

The point is that we can postulate a time spectrum for the durability of assets. On one extreme, reflecting a very high degree of durability, we would find assets such as real estate. Near the other polar extreme on that spectrum, we would find assets of a much shorter duration such as stock options. But even closer to that opposite pole we would find sports bets.

3 THE GENERALIZED POISSON BINOMIAL DISTRIBUTION

The generalized Poisson binomial (GPB) distribution was introduced by Zhang et al. (2018). The distribution posits a binary response, U_i, for event i, with values A_i and B_i for each outcome with probability p_i. and $(1 - p_i)$, respectively. The GPB variable is then defined as $U = \Sigma_i U_i$. When all $A_i = 1$ and all $B_i = 0$, the model reduces to the standard Poisson-Binomial distribution.

The sports gambling environment often follows the GPB. Many sports betting environments (baseball, football, ba sketball, prize fighting, etc.,) involve a two-party relationship between teams or individual contenders, meaning that there are only two opposing teams with only two outcomes. This environment would not apply to a soccer contest when there are three possible outcomes (win, draw, or loss). This is to be contrasted with events in which there are greater than two outcomes such as horse racing or golf tournaments. For two-party sports, then, there is a known payoff when winning and a known loss when losing. Additionally, probabilities p_i. and $(1 - p_i)$ can often be estimated.

For any contest, i, define X_i and Y_i to be the amount bet on the favourite and the dog, respectively. Let a_i be the gain per dollar bet on the favourite, if the favourite wins. Let c_i be the gain per dollar bet on the dog, if the dog wins. Let d_i be the loss per dollar bet on the dog, if the favourite wins. Let b_i be the loss per dollar bet on the favourite if the dog wins. (Generally, $b_i = -1$ and $d_i = -1$ unless a premium is offered as part of the bet.) Thus, the return, $r_i = a_iX_i + d_iY_i = A_i$ with probability p_i if the favourite wins and $r_i = b_iX_i + c_iY_i = B_i$ with probability $(1 - p_i)$ if the dog wins. It is easily seen that the expected return and the variance of r_i are as follows:

$$\mu_i = \left[b_i + (a_i - b_i)p_i\right]X_i + \left[d_i + (c_i - d_i)(1 - p_i)\right]Y_i \quad (1a)$$

$$\sigma_i^2 = \left[(a_i - b_i)X_i + (c_i - d_i)Y_i\right]^2 p_i(1 - p_i). \quad (1b)$$

Zhang et al. (2018) presented two ways to calculate the probability of U, an exact method and an approximate method. The exact method will be discussed and utilized later for the Post-Modern Portfolio Theory analysis. Thus, the GPB distribution well models the commonly occurring two contender contests and provides a unique approach to developing portfolios of such contests for the investment-driven bettor.

4 THE MODERN PORTFOLIO THEORY ANALYSIS

This approach, introduced by Markowitz (1952), maximizes expected return for a given level of risk. Prior to the introduction of the theory, there were a few attempts to discuss risk and diversification in terms of assets. Hicks (1935) introduced risk into his demand of money model by discussing an approach to better understand the valuation of assets under uncertainty. “It is convenient to represent these probabilities to oneself, in statistical fashion, by a mean value, and some appropriate measure of dispersion” (Hicks 1935, p. 8). He assured that investing in a large number of securities could nearly eliminate risk and introduced a way to value a security in an uncertain environment. Williams (1938, p. 67) added, “whenever the value of a security is uncertain and has to be expressed in terms of probability, the correct value to choose is the mean value.” Leavens (1945) noted that, when independent factors act on a security, diversification has benefits, but, in practice, this is hard to achieve, and that diversification alone cannot protect against cyclical factors. Modern Portfolio Theory pioneered by Markowitz (1952) introduces portfolio management built upon a mean-variance model and assumes a normal distribution of returns.

Applying this approach to sports betting leads to the construction of a quadratic programming problem as follows:

The method constructs a quadratic programming problem as follows:

$$\text{Max}_{(X_i, Y_i)}\ \Sigma_i[\mu_i - K\sigma_i^2] \qquad (2)$$

$$st. \sum_i \left(X_i + Y_i\right) = D \text{ and } X_i, Y_i, K \geq 0, \text{where } D = \text{money bet}.$$

The variable K controls the level of risk, where risk is measured by the variance. Thus, for given values of a_i, b_i, c_i, and d_i along with p_i using equation (1), and setting the level of risk, K, the quadratic programming problem identifies the portfolio that maximizes the expected return. By varying K, and repeatedly solving the problem, an efficient frontier can be developed.

EXAMPLE: ANALYSING MARCH MADNESS USING MPT

March Madness is a high-profile college basketball tournament in the United States, which began in 1939 and generally played in March. As a result of the COVID-19 virus, it was cancelled in 2020, and the tournament in 2019 was used as a source of data. The tournament consists of several rounds or levels of play. For simplicity, this discussion will focus on one specific round, the Elite Eight round, which consists of the final eight teams and therefore four contests. While the examples here are all based on March Madness, there is no reason why other two-party contests, from other sports, could not be analysed in a similar manner.

For these examples, the betting payoffs were taken from the Betonline (www.betonline.ag) site. The probabilities were taken from the 538 (fivethirtyeight.com) site. 538's March Madness forecasts are constructed of 75% computer ratings (Basketball Power Index, Pomeroy, Sagarin, Moore, Sokol, and Elo ratings) and 25% human rankings (NCAA S-Curve and Preseason rankings from the AP and coaches). These outputs are adjusted each round for injuries, and bonuses are awarded for the quality of the opponent and score. In both cases, these values were taken just prior to the first Elite Eight contest. The various payoffs and actual outcomes may be found in the Appendix**.** Note that five of the seven optimal solutions (excluding the initial, subjective bet) yielded a positive return, suggesting that thoughtful betting (intelligent investing) may, indeed, be profitable. The initial subjective solution yielded a negative return.

There are various ways to develop a bettor's attitude towards risk. One simple approach is to ask an investment-driven bettor to thoughtfully develop an initial bet. That approach was used here. In each of the examples that follow, the amount of the bet is D = \$100. Table 1 shows the initial bet for the Elite Eight teams that competed in 2019.

The above bet was evaluated for the Elite Eight contest, using equations (1a) and (1b). This initial bet had an expected return of −\$6.27 and a variance (risk) of \$2,357.91. The MPT optimization was then performed with the added constraint that the variance is equal to \$2,357.91 as shown in Table 2.

Table 1: Initial bet

X1	Y1	X2	Y2	X3	Y3	X4	Y4
Duke	Michigan State	Gonzaga	Texas Tech	Virginia	Purdue	Kentucky	Auburn
25.00	0.00	0.00	15.00	0.00	20.00	40.00	0.00

Table 2. MPT constrained to initial bet variance

X1	Y1	X2	Y2	X3	Y3	X4	Y4
Duke	Michigan State	Gonzaga	Texas Tech	Virginia	Purdue	Kentucky	Auburn
33.31	0.00	0.00	7.55	19.98	10.72	0.00	28.43

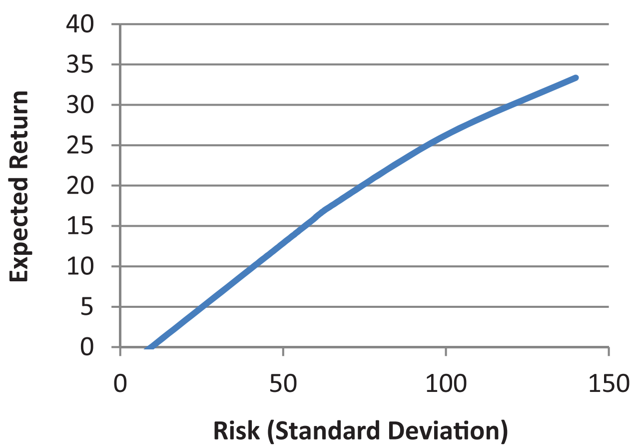

Figure 1. Expected return for MPT

The new value of the expected return was $12.41 and the variance of the new bet, of course, was unchanged.

The efficient frontier can be seen in Figure 1.

5 THE POST-MODERN PORTFOLIO THEORY

Post-Modern Portfolio Theory is a collection of literature that seeks to improve upon Markowitz's original theory. The improvements often focus on downside risk as opposed to variance and suggest that returns are not normally distributed. Sharpe (1964, p. 428) argued that Markowitz suggested that "semi-variance would be preferable." Peter Fishburn (1977) introduced a model to capture the magnitude effects of downside risk for the individual investor. Aitchison and Brown (1957) introduced the lognormal distribution that better mirrored the returns of the market. Efron and Tibshirani (1993) introduced the bootstrap procedure. Sortino and van der Meer (1991) popularized the use of downside risk and argued that often the theory produced better results than a mean-variance optimizer. Rom and Ferguson (1993) introduced the name of post-modern portfolio theory in their paper Modern Portfolio Theory Comes of Age.

EXAMPLE: ANALYSING MARCH MADNESS USING PMPT

The PMPT approach to the betting portfolio environment is centred around the measurement and control of the downside risk. The following problem formulation is developed to measure and control both the upside and downside risk in the optimal portfolios. The approach is based on the exact method described by Zhang et al. (2018). The M matrix will be defined as a binary matrix that clocks through all values starting with all zeros in the first row and all ones in the last. For the Elite Eight contest, there are four contests, so M contains $2^4 = 16$ rows. Matrix M is displayed in Table 3.

Let m_{ki} = the typical element of M. Then define the return and probability components as follows:

$$\begin{aligned} &r_{ki} = A_i = A(X_i, Y_i) \quad \text{if } m_{ki} = 0 \text{ and} \\ &r_{ki} = B_i = B(X_i, Y_i) \quad \text{if } m_{ki} = 1 \text{ and} \\ &p_{ki} = p_i \quad \text{if } m_{ki} = 0 \text{ and} \\ &p_{ki} = (1 - p_i) \quad \text{if } m_{ki} = 1. \\ &\text{with } r_k = \sum_i r_{ki} \text{ and } p_k = \pi_i p_{ki} \end{aligned} \tag{3}$$

Denote the minimum acceptable risk (MAR) as T. Then the expected return of the portfolio of bets is then $\mu = \Sigma_k r_k p_k$ and the risk is $\rho = \Sigma_k (T - r_k)^2 p_k$. It should be noted that the variance of the portfolio of bets, $V(r_k) = \Sigma_k (r_k - \mu_k)^2 p_k$, can be computed as ρ–. Downside risk is defined as $\Sigma_{rk < T}(T - r_k)^2$ and upside risk is defined as $\Sigma_{rk \geq T}(T - r_k)^2$. The following quadratic-binary programming formulation then maximizes the expected return subject to weights on the downside and upside risks:

$$\begin{aligned} &Max_{(X_i, Y_i)} \left\{ \sum_k E[r_k] - K_1 \sum_{rk<T} (T - r_k)^2 z_k - K_2 \sum_{rk \geq T} (T - r_k)^2 (1 - z_k) \right\} \\ &\text{St } \sum_i (X_i + Y_i) = D \\ &T(2z_k - 1) \geq r(2z_k - 1) \end{aligned} \tag{4}$$

and X_i, $Y_i \geq 0$, z_k is binary, where D = money bet and T = minimum acceptable return (*MAR*)

Table 3. The M matrix

0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

It is of interest that this formulation isolates the downside risk separately from the upside risk. As K_1 and K_2, the weights for the downside and upside risks, respectively, are unrestricted, the bettor is completely free to weigh either kind of risk as desired.

Now using the formulas for the expected values and risk, the initial bet has an expected value of −6.27 and a downside and upside risk of 1,934.06 and 688.42, respectively; here $T = 10$.[1] When these risk values are held constant at those values in the optimization routine, the new bet is shown in Table 4.

Here the expected return is $6.12. However, the fixing of the upside risk is problematic as restrictions on upside risk are generally not warranted. When this restriction is removed, the new solution is seen in Table 5.

The expected return is $17.09. The efficient frontier with $K_2 = 0$ and varying K_1 is shown in Figure 2.

6 A COMMENT ON THE KELLY CRITERION

The Kelly criterion as proposed by Kelly (1956) was developed based upon the geometric mean and thus led to a geometric mean-variance criterion as opposed to an arithmetic mean-variance model which was the basis of the Markowitz model (Kim and Shin, 2017). The method, when

[1] As stated above, $V(r_k) - \rho-$ so that for the initial bet: 2,357.91 = 2,622.48 – $(10 + 6.27)^2$, with slight roundoff error.

the outcome of each bet is two-party with "well defined odds and probabilities, and successive bets are mutually independent," results in a "simple formula for the critical fraction of wealth that will maximize a gambler's average return over a large number of bets" (Peterson 2018, p. 15). The model is known among gamblers and has been the subject of a number of papers. *The Kelly Criterion in Blackjack, Sports Betting and the Stock Market* by Thorp (2006) introduces the criterion to determine the bet size of each matchup. Baker and McHale (2013) introduced an improvement to the model in betting situations which improves out-of-sample performance by shrinking the size of the bet in the presence of the parameter uncertainty. Haigh (2000) used the criterion in a spread betting environment and found that the goal "of aiming to maximize the long-term capital growth rate has attractions, but it may turn out to be impractical or to indicate bets that are larger than a gambler is willing to make" (Haigh, 2000 p. 539). Grauer (1981) found that the expected return and standard deviation of unleveraged portfolios constructed using geometric mean optimization were higher than those of mean value

Table 4. PMPT constrained to initial bet risks

X1	Y1	X2	Y2	X3	Y3	X4	Y4
Duke	Michigan State	Gonzaga	Texas Tech	Virginia	Purdue	Kentucky	Auburn
65.90	0.00	0.00	0.36	20.80	11.57	0.00	1.36

Table 5. PMPT constrained to initial bet downside risk

X1	Y1	X2	Y2	X3	Y3	X4	Y4
Duke	Michigan State	Gonzaga	Texas Tech	Virginia	Purdue	Kentucky	Auburn
62.05	0.00	0.00	2.73	0.00	2.92	0.00	32.3

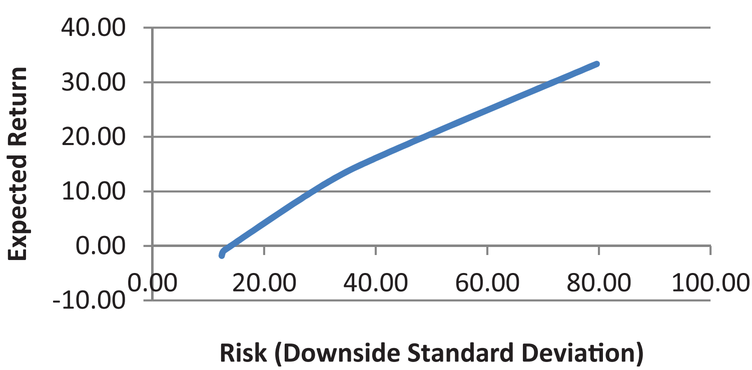

Figure 2. Expected return for PMPT

optimization. Kim and Shin (2017) reported similar results. Grauer (1981) concluded that geometric portfolios were inefficient compared with the mean value portfolios. Thus, there has been a wide variety of results reported for the Kelly criterion. The overarching argument for the Kelly criterion has been its long-term optimality. However, because of the generally short duration of sports betting, the benefit of such optimality is questionable.

EXAMPLE: THE KELLY CRITERION

The Kelly criterion was applied to the March Madness Elite Eight data. The method used closely follows Thorp's (2006) discussion of two simultaneous bets. If the betting fraction is f, and a one-dollar bet is lost, the value of the resulting wealth is $1 - f$ and if the bet is won, the value of the resulting wealth is $1 + wf$, where w are the winnings per dollar bet.

For the example, let $x_i = X_i/D$ and $y_i = Y_i/D$ be the fractional bets on the favourite and the dog for contest i, respectively, and D is the money bet. Then note that m_{ki} is the typical element of M, with $m_{ki} = 0$ if the favourite wins and $m_{ki} = 1$ if the favourite loses. Here are the winning and losing amounts:

$$\begin{aligned} r_{x,ki} &= -x_i && \text{if } m_{ki} = 1 \text{ and} \\ r_{x,ki} &= a_i x_i && \text{if } m_{ki} = 0 \text{ and} \\ r_{y,ki} &= -y_i && \text{if } m_{ki} = 0 \text{ and} \\ r_{y,ki} &= c_i y_i && \text{if } m_{ki} = 1. \end{aligned} \tag{5}$$

There are 16 rows to M, yielding 16 values of p_k probabilities, as defined in equations (3). The following nonlinear programming problem yields the Kelly optimum:

$$\begin{aligned} &Max_{(xi,yi)} [\Sigma_k p_k \Sigma_i \ln(1 + \Sigma_i(r_{x,ki} + r_{y,ki}))] \\ &st.\ \Sigma_i (x_i + y_i) = 1 \\ &and\ x_i, y_i \geq 0. \end{aligned} \tag{6}$$

Then $X_i = D\,x_i$ and $Y_i = D\,y_i$.

The Kelly solution is easily obtained using Solver in MS Excel. The Kelly optimal solutions bets are shown in Table 6.

This solution has an expected value of $6.87 and a variance of $985.80 and, with T = 10, the downside and upside risks are $571.32 and $424.26, respectively.

To compare this solution with the MPT analysis, the MPT problem is constrained with a variance of $985.80, to yield an expected return of $6.95. That solution for the Elite Eight teams is shown in Table 7.

Table 6. Kelly solution

X1	Y1	X2	Y2	X3	Y3	X4	Y4
Duke	Michigan State	Gonzaga	Texas Tech	Virginia	Purdue	Kentucky	Auburn
21.94	2.41	21.59	15.36	12.40	6.66	0.00	19.46

Table 7. MPT constrained to Kelly variance

X1	Y1	X2	Y2	X3	Y3	X4	Y4
Duke	Michigan State	Gonzaga	Texas Tech	Virginia	Purdue	Kentucky	Auburn
21.53	0.00	0.00	4.88	35.62	19.58	0.00	18.39

Table 8. PMPT constrained to Kelly downside risk

X1	Y1	X2	Y2	X3	Y3	X4	Y4
Duke	Michigan State	Gonzaga	Texas Tech	Virginia	Purdue	Kentucky	Auburn
21.94	2.41	21.59	15.36	12.40	6.66	0.00	19.46

Table 9. Kelly constrained to initial bet downside risk

X1	Y1	X2	Y2	X3	Y3	X4	Y4
Duke	Michigan State	Gonzaga	Texas Tech	Virginia	Purdue	Kentucky	Auburn
17.30	5.18	16.16	10.86	0.34	0.00	0.00	50.16

Thus, in this case, the Kelly criterion performs near the MPT efficient frontier value, for the Kelly level of risk, although the distribution of bets is somewhat different.

If the PMPT is calculated at the Kelly level of downside risk, then the solution can be seen in Table 8.

Interestingly, this solution is the same as the Kelly solution. It is to be noted that both the MPT and the PMPT solutions, when constrained to the Kelly variance and downside risks, respectively, are at or near the Kelly solution.

If the downside risk is held constant at the level of the initial bet, then the new Kelly bet for each of the Elite Eight teams can be seen in Table 9.

The expected return for this solution is \$16.74 and the upside risk is \$3,105.78, which is near the PMPT solution of \$17.09, with no constraint on the upside risk.

In this example, for the optimization results, the Kelly criterion did not outperform the MPT or the PMPT methods presented, although it performed nearly as well. The MPT and PMPT approaches have the advantage that they provide Efficient Frontier portfolios for any desired level of risk, whereas the Kelly solution yields only one level of risk. On the contrary, the argument for the Kelly approach is that it claims to have optimal long-term wealth properties. However, it must be noted that only the Elite Eight March Madness for 2019 has been analysed. It is not clear how generalizable the results of these examples are, and the presumed long-term benefits of the Kelly approach cannot be determined from these examples.

7 MARKET COMPARISON

Using S&P data as a surrogate index for the Market, from April 2009 to March 2019, the mean monthly return on the index was 0.0106 with a standard deviation of 0.0359. Using the coefficient of variation (COV) as a normalized risk for the S&P, the COV was 3.395. The mean return for the MPT optimal solution was 0.1241 with a standard deviation of 0.4856 yielding a COV of 3.913. The ratio of the COV for the MPT optimal solution to the COV of the S&P index was 1.153. That is, the increase in the normalized risk (COV) of the MPT optimal solution, as compared with the Market, was about 15%. When comparing the results with the Nasdaq composite index, the mean monthly return on the index from April 2009 to March 2019 was 0.0136 with a standard deviation of 0.043 yielding a COV of 3.13. The ratio of the optimal portfolio COV to the Nasdaq is 1.251. The Russell 2000 provides a relatively unique comparison as it had a mean monthly return from April 2009 to March 2019 of 0.0109 and a standard deviation of 0.05 which yields a COV of 4.54. The ratio of the optimal portfolio COV to the Russell 2000 COV is 0.862. As the Market is presumed to be on the efficient frontier, it can be expected that many actual investments will be non-optimal with a higher risk for any given level of return. Said another way, the MPT optimal solution appears competitive in normalized risk to the risks associated with the Market.

Now it is speculated here that the reason the MPT optimal solution is competitive has to do with the economic dynamics associated with the sports betting process. It is hypothesized that the average entertainment-driven bettor will make substantially non-optimal bets. This provides a large potential profit for the organization (house) offering the bets. In an attempt to increase the profit for the house, by increasing the number of bettors and the amount of the bets, the house will offer more favourable odds. The tendency will be amplified by the degree of competitive forces among sports betting houses in the betting market. Eventually, assuming a high degree of competition among the betting houses, all the sports betting houses will settle at competitive market odds. However, this process does not increase the quality of the bets for the entertainment-driven bettor. So, this process leads to better odds, but not better performance by the entertainment-driven bettor. However, the investment-driven bettor, who now sees more favourable odds, can expect risk characteristics of the (optimal) bets to be favourable, as suggested by the examples above, and can expect a favourable return.

While the normalized risks associated with the optimal sports bet may be similar to that of the market, there are some significant differences. The optimal sports bet is much riskier. The expected return of 12.4 requires is associated with a risk of 48.8. If an expected return of 12.4 is desired from the S&P, an investment of 12.4/0.0106 = $1,171.54 (approximately). The associated risk would be 12.4*3.395 = 42.10, which is slightly less than that of the sports bet. Thus, a much greater investment in the Market is needed to expect a comparable return, with a comparable risk. It should be noted that a major advantage of the optimal sports bet is that the return can be realized in a much shorter time period.

While the research described here is limited to a single example, namely the March Madness Elite Eight contest, it is suggestive that, for the investment-driven bettor, serious investing in sports betting is worthy of consideration.

8 CONCLUSIONS

This paper focuses on three ideas. The first is that sports betting should be considered an asset class. The second is that the two-party sports betting environment can be well described as a generalized Poisson binomial distribution. The third is that the MPT, the PMPT, and the Kelly criterion analyses may be performed using the generalized Poisson binomial distribution results. The MPT leads to a quadratic programming problem, the PMPT leads to a unique quadratic-binary programming problem, and the Kelly criterion leads to a nonlinear programming problem.

It is worth noting that these results are generalizable to two-party intelligent sports betting, but not non-two-party betting (such as a wager on the winner of a golf tournament, a horse race, or a downhill skiing competition). Also, since sports betting is viewed as an asset class, in comparing betting schemes, the approaches described here not only consider expected return but also total risk as well as upside and downside risk. Risk should be a consideration for the investment-driven bettor.

There are a number of possible extensions suggested by this research. Integrating several different sports or competitions within a sport at different levels (i.e., college and NBA basketball) into the model, thereby increasing diversification is easily accomplished in this model. Other extensions of this research include integrating the sports betting optimal portfolio with more traditional financial investment portfolios (further increasing the potential for diversification), developing portfolios that are simultaneously optimal for all levels of a sequential contest, that is, contests with several levels of play (such as March Madness), and the use of results from an earlier level of play to update forecasts at a later level of play. Certainly, many other extensions are possible.

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APPENDIX

Here are the probabilities and the payoffs for the bets on Elite Eight. They should be understood as a single realization of the generalized Poisson binomial distribution for the betting process and interpreted appropriately. The probabilities and the payoffs are presented in Table A1. The contest winner is shown with a double asterisk (**).

Table A1. Probabilities and payoffs used in analyses

Favourites	**Duke**	**Gonzaga**	**Virginia****	**Kentucky**
Prob	0.67	0.63	0.64	0.52
a	$0.65	$0.45	$0.51	$0.48
b	–$1.00	–$1.00	–$1.00	–$1.00
Dogs	**Mich St****	**Texas Tech****	**Purdue**	**Auburn****
Prob	0.33	0.37	0.36	0.48
c	$1.35	$1.90	$1.71	$1.80
d	–$1.00	–$1.00	–$1.00	–$1.00

The actual outcome profits and losses are presented in Table A2.

Table A2. Actual profits and losses of solutions

Profit	**Solutions in Order Presented in Paper**
–$56.50	Initial Bet Solution
$31.68	MPT Constrained to Initial Bet Variance
–$63.73	PMPT Constrained to Initial Bet Risks
–$1.64	PMPT Constrained to Initial Bet Downside Risk
$23.59	Kelly Solution
$19.44	MPT Constrained to Kelly Bet Variance
$23.59	PMPT Constrained to Kelly Downside Risk
$84.63	Kelly Constrained to Initial Bet Downside Risk